

Lecture 1

Thévenin Equivalent Circuits

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Thévenin Equivalent Circuits

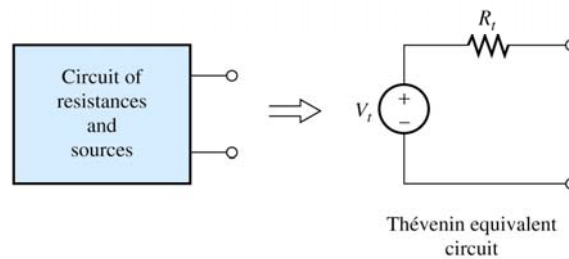


Figure 2.40 A two-terminal circuit consisting of resistances and sources can be replaced by a Thévenin equivalent circuit.

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Thévenin Equivalent Circuits

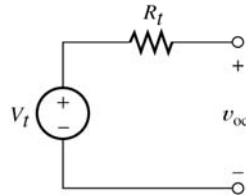


Figure 2.41 Thévenin equivalent circuit with open-circuited terminals. The open-circuit voltage v_{oc} is equal to the Thévenin voltage V_t .

$$V_t = v_{oc}$$

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Thévenin Equivalent Circuits

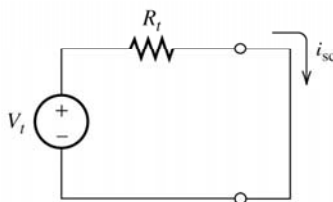
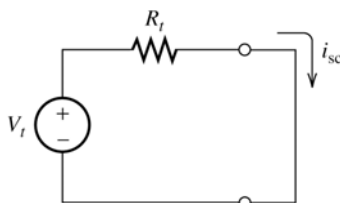


Figure 2.42 Thévenin equivalent circuit with short-circuited terminals. The short-circuit current is $i_{sc} = V_t/R_t$.

$$i_{sc} = \frac{V_t}{R_t}$$

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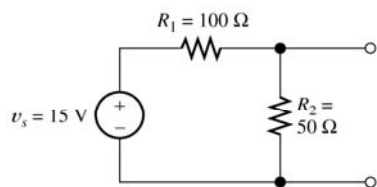
Thévenin Equivalent Circuits



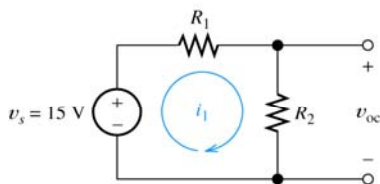
$$R_t = \frac{V_{oc}}{i_{sc}}$$

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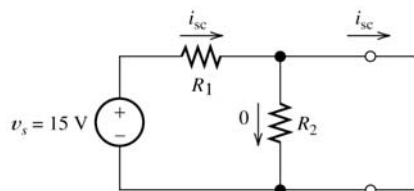
Find the Thevenin Equivalent Circuit



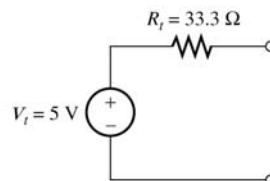
(a) Original circuit



(b) Analysis with an open circuit



(c) Analysis with a short circuit

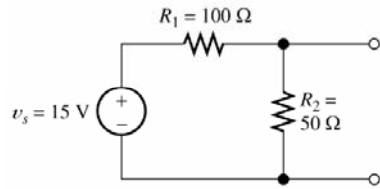


(d) Thévenin equivalent

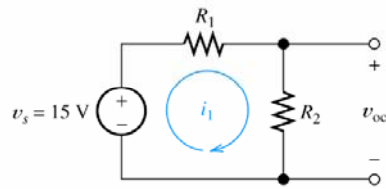
Figure 2.43 Circuit for Example 2.14.

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Find the Thevenin Equivalent Circuit



(a) Original circuit



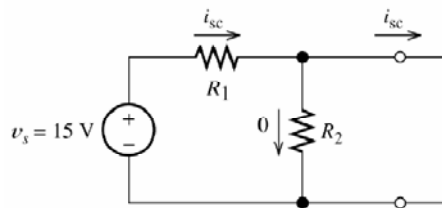
(b) Analysis with an open circuit

Use voltage divider formed by R_1 and R_2 to find v_{oc} :

$$v_{oc} = \frac{R_2}{R_1 + R_2} v_s = \frac{50}{100 + 50} 15V = 5V$$

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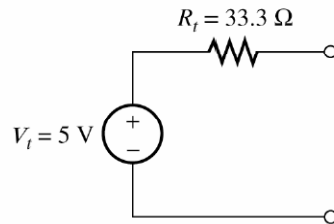
Find the Thevenin Equivalent Circuit



$$i_{sc} = \frac{v_s}{R_1} = \frac{15V}{100\Omega} = 0.15A$$

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Find the Thevenin Equivalent Circuit



$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{5V}{0.15A} = 33.3\Omega$$

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Find the Thevenin Equivalent Circuit

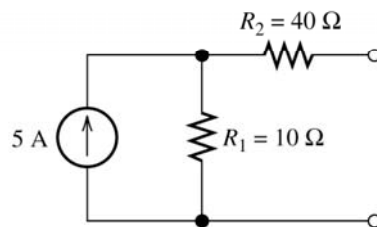
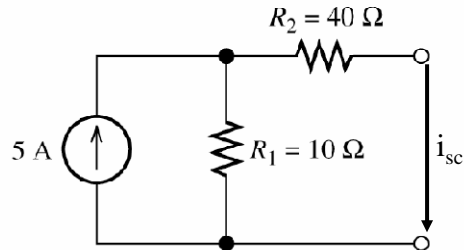


Figure 2.44 Circuit for Exercise 2.22.

$$v_{oc} = (5A)(10\Omega) = 50V$$

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Find the Thevenin Equivalent Circuit



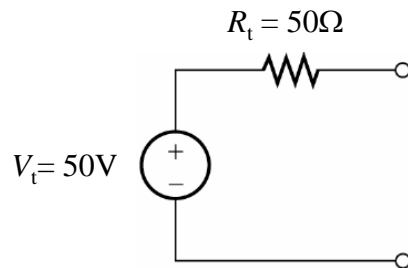
Use current divider to find i_{sc} :

$$i_{sc} = \frac{R_1}{R_1 + R_2} 5A = \frac{10}{10 + 40} 5A = 1A$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{50V}{1A} = 50\Omega$$

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Find the Thevenin Equivalent Circuit



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Finding the Thévenin Resistance Directly

We can find the Thévenin resistance by zeroing the sources in the original network and then computing the resistance between the terminals.

When zeroing a voltage source, it becomes a short circuit. When zeroing a current source, it becomes an open circuit.

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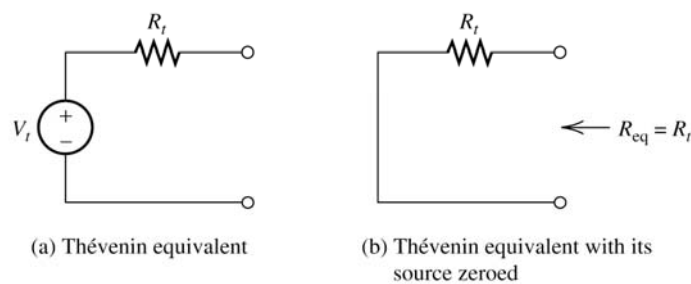


Figure 2.45 When the source is zeroed, the resistance seen from the circuit terminals is equal to the Thévenin resistance.

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Find the Thevenin Equivalent

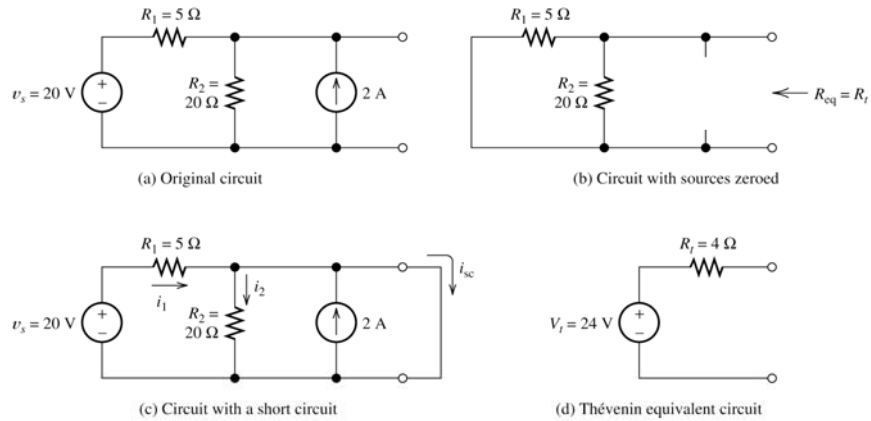
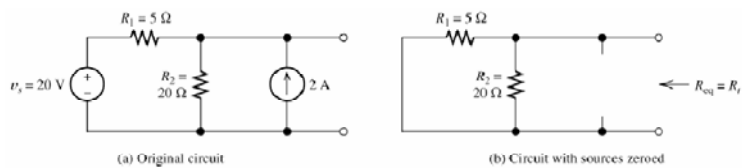


Figure 2.46 Circuit for Example 2.15.

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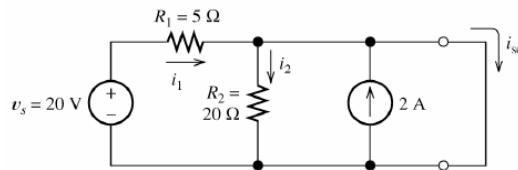
Find the Thevenin Equivalent



$$R_{eq} = R_t = \frac{R_1 R_2}{R_1 + R_2} = \frac{100}{25} = 4\Omega$$

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Find the Thevenin Equivalent



(c) Circuit with a short circuit

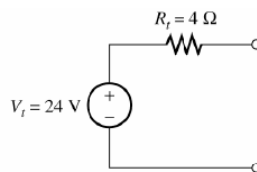
$$i_2 = 0$$

$$i_1 = \frac{20V}{5\Omega} = 4A$$

$$i_{sc} = i_1 + 2A = 4A + 2A = 6A$$

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Find the Thevenin Equivalent



$$V_t = R_t i_{sc} = (4\Omega)(6A) = 24V$$

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Find the Thevenin Resistance

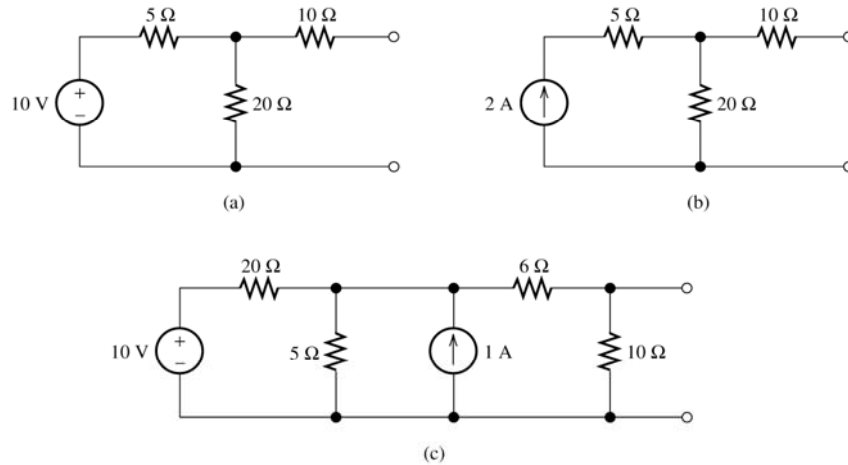
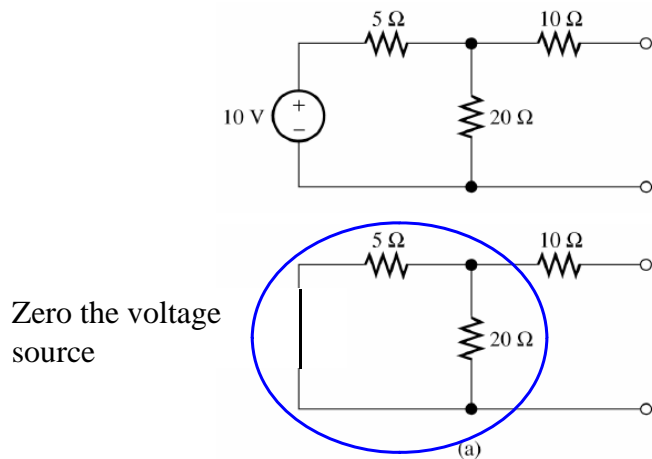


Figure 2.47 Circuits for Exercise 2.24.

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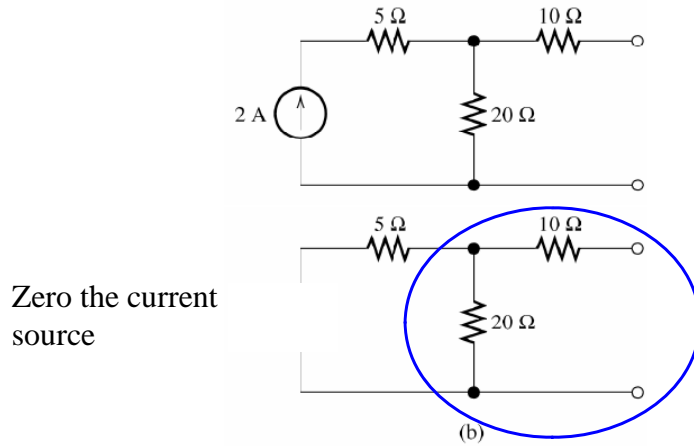
Find the Thevenin Resistance



$$R_t = 10 + \frac{(5)(20)}{5 + 20} = 10 + 4 = 14\Omega$$

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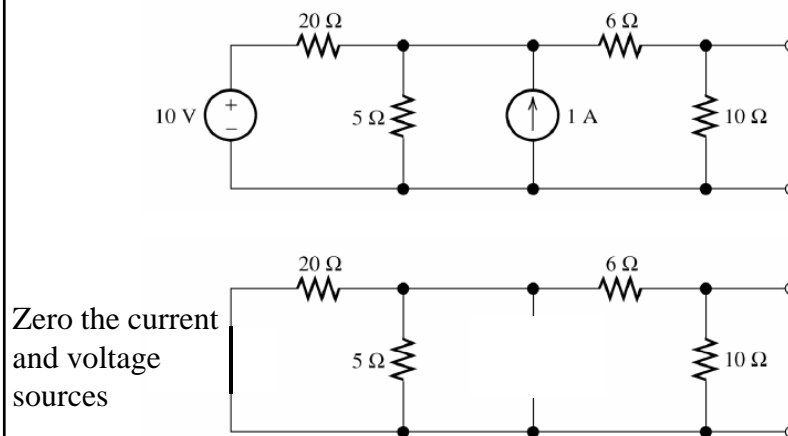
Find the Thevenin Resistance



$$R_t = 10 + 20 = 30\Omega$$

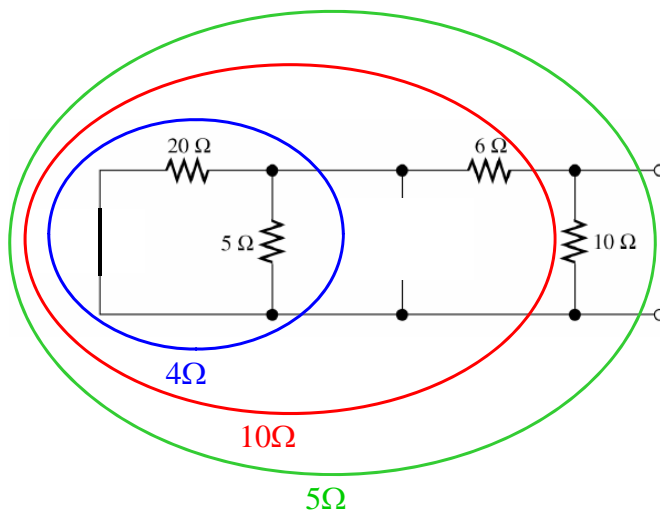
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Find the Thevenin Resistance

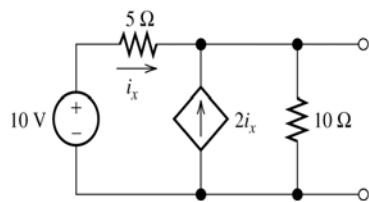


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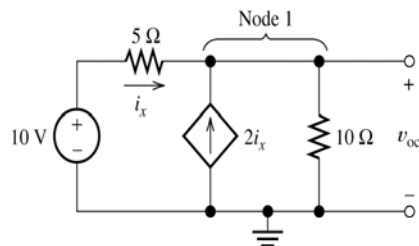
Find the Thevenin Resistance



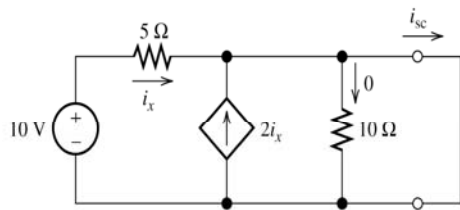
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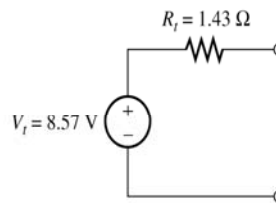
(a) Original circuit



(b) Circuit with an open circuit



(c) Circuit with a short circuit

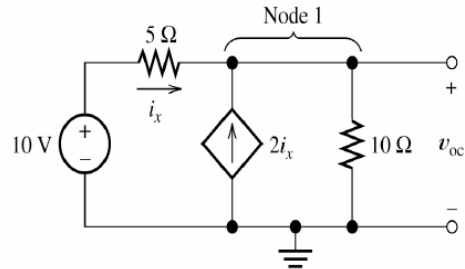


(d) Thévenin equivalent

Figure 2.48 Circuit for Example 2.16.

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Thevenin Equivalent with a Dependent Source



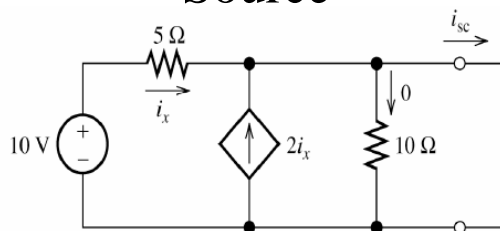
(b) Circuit with an open circuit

$$KCL \text{ at Node 1: } i_x + 2i_x = \frac{v_{oc}}{10} = 3i_x$$

$$i_x = \frac{10 - v_{oc}}{5} \rightarrow 3 \frac{10 - v_{oc}}{5} = \frac{v_{oc}}{10} \rightarrow v_{oc} = 8.57V$$

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Thevenin Equivalent with a Dependent Source



(c) Circuit with a short circuit

$$i_x + 2i_x = 3i_x = i_{sc}$$

$$i_x = \frac{10V}{5\Omega} = 2A \rightarrow i_{sc} = 6A$$

$$R_t = \frac{V_t}{i_{sc}} = \frac{8.57V}{6A} = 1.43\Omega$$

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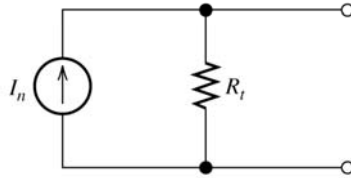


Figure 2.49 The Norton equivalent circuit consists of an independent current source I_n in parallel with the Thévenin resistance R_t .

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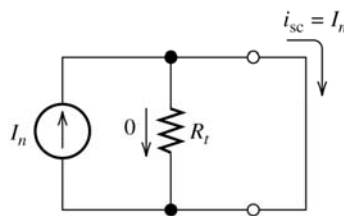


Figure 2.50 The Norton equivalent circuit with a short circuit across its terminals.

$$I_n = i_{sc}$$

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Step-by-step Thévenin/Norton-Equivalent- Circuit Analysis

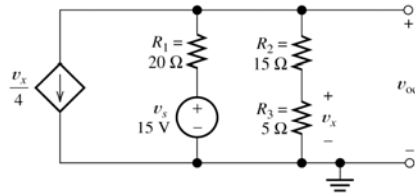
1. Perform two of these:
 - a. Determine the open-circuit voltage $V_t = v_{oc}$.
 - b. Determine the short-circuit current $I_n = i_{sc}$.
 - c. Zero the sources and find the Thévenin resistance R_t looking back into the terminals.

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2. Use the equation $V_t = R_t I_n$ to compute the remaining value.
3. The Thévenin equivalent consists of a voltage source V_t in series with R_t .
4. The Norton equivalent consists of a current source I_n in parallel with R_t .

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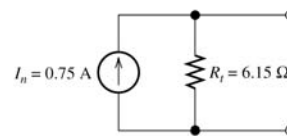
Find the Norton Equivalent



(a) Original circuit under open-circuit conditions



(b) Circuit with a short circuit

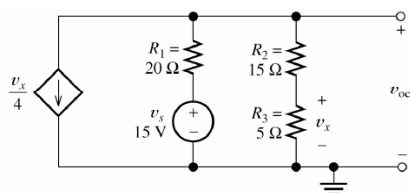


(c) Norton equivalent circuit

Figure 2.51 Circuit of Example 2.17.

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Find the Norton Equivalent



(a) Original circuit under open-circuit conditions

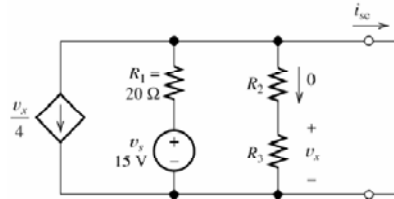
$$KCL: \frac{v_x}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0$$

$$\text{Voltage divider: } v_x = \frac{R_3}{R_2 + R_3} v_{oc} = 0.25v_{oc}$$

$$\frac{0.25v_{oc}}{4} + \frac{v_{oc} - 15}{R_1} + \frac{v_{oc}}{R_2 + R_3} = 0 \rightarrow v_{oc} = 4.62V$$

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Find the Norton Equivalent



(b) Circuit with a short circuit

$$v_x = 0$$

$$i_{sc} R_1 - 15V = 0 \rightarrow i_{sc} = \frac{15V}{20\Omega} = 0.75A$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{4.62V}{0.75A} = 6.15\Omega$$

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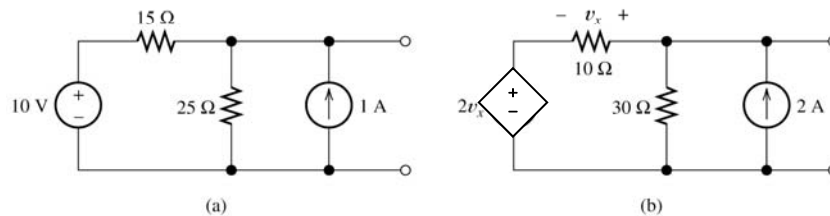
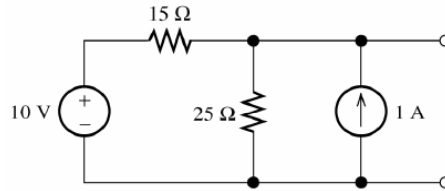


Figure 2.52 Circuits for Exercise 2.25.

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Find the Norton Equivalent

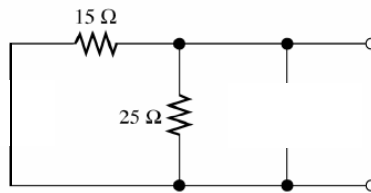


(a)

$$\frac{v_{oc} - 10}{15} + \frac{v_{oc}}{25} - 1 = 0 \rightarrow v_{oc} = \frac{125}{8} = 15.63A$$

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Find the Norton Equivalent



(a)

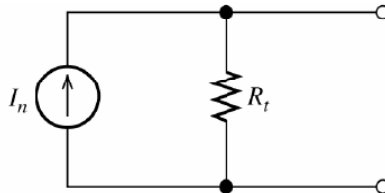
To find R_t zero the sources:

$$R_t = \frac{(15\Omega)(25\Omega)}{15\Omega + 25\Omega} = 9.38\Omega$$

$$i_{sc} = \frac{V_t}{R_t} = \frac{15.63V}{9.38\Omega} = 1.67A$$

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Find the Norton Equivalent

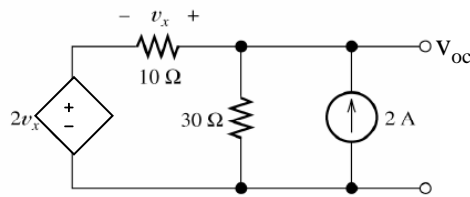


$$R_t = 9.38\Omega$$

$$i_n = 1.67A$$

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Find the Norton Equivalent



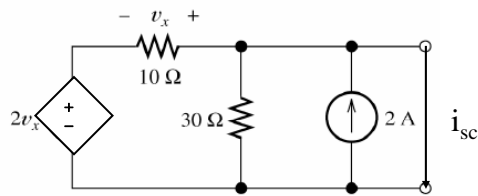
(b)

$$KVL: v_{oc} - 2v_x - v_x = 0 \rightarrow v_{oc} = 3v_x$$

$$KCL: \frac{v_{oc} - 2v_x}{10} + \frac{v_{oc}}{30} - 2 = 0 \rightarrow v_{oc} = 30V$$

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Find the Norton Equivalent



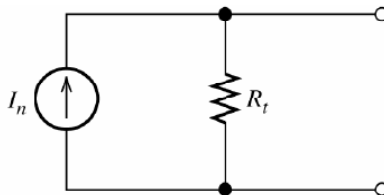
(b)

$$KVL: -2v_x - v_x = 0 \rightarrow v_x = 0$$

$$KCL: \frac{0 - 2v_x}{10} + i_{sc} - 2 = 0 \rightarrow i_{sc} = 2A = i_n$$

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Find the Norton Equivalent



$$i_n = 2A$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{30V}{2A} = 15\Omega$$

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Source Transformations

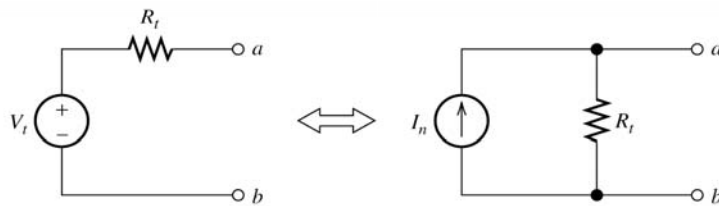


Figure 2.53 A voltage source in series with a resistance is externally equivalent to a current source in parallel with the resistance, provided that $I_n = V_t/R_t$.

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Source Transformations

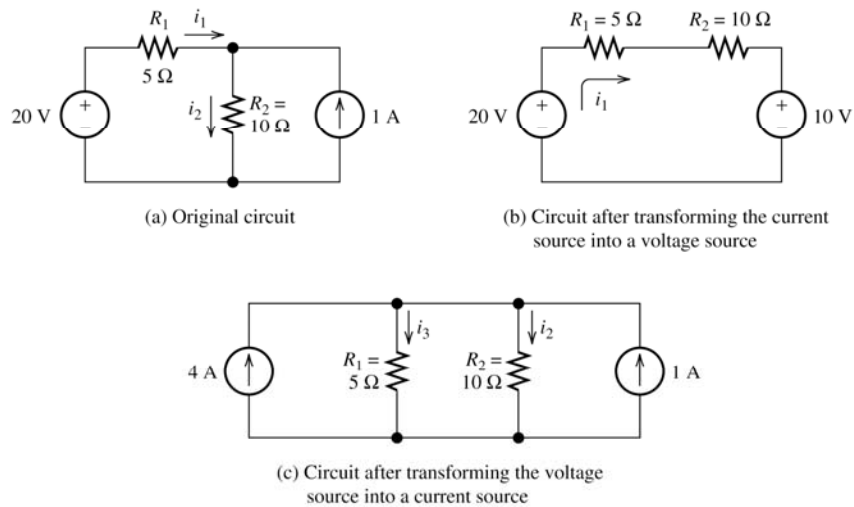
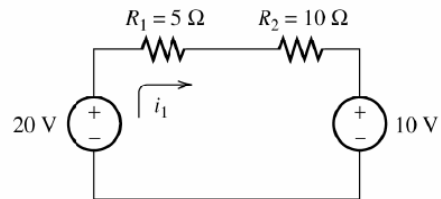


Figure 2.54 Circuit for Example 2.18.

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Source Transformations



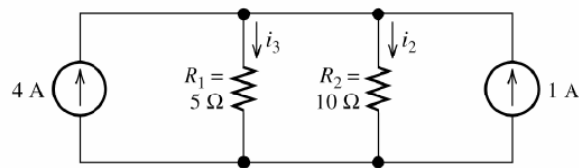
(b) Circuit after transforming the current source into a voltage source

$$V_t = i_n R_t = (1A)(10\Omega) = 10V$$

$$R_1 i_1 + R_2 i_1 + 10 - 20 = 0 \rightarrow i_1 = \frac{10}{R_1 + R_2} = 0.667A$$

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Source Transformations



(c) Circuit after transforming the voltage source into a current source

$$i_n = \frac{V_t}{R_t} = \frac{20V}{5\Omega} = 4A$$

$$i_2 + i_3 = 5A$$

$$\text{Current divider: } i_2 = \frac{R_1}{R_1 + R_2} 5A = \frac{5}{5 + 10} 5A = 1.667A$$

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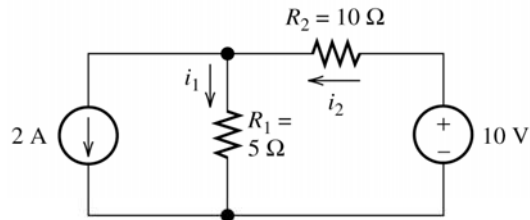
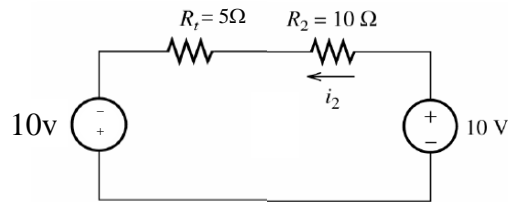


Figure 2.55 Circuit for Exercise 2.26.

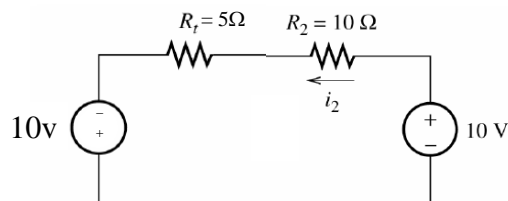
Transform the Norton equivalent into a Thevenin equivalent with $V_t = (2A)(5\Omega) = 10V$, $R_t = 5\Omega$

Note the reference direction for the current gives the polarity for the voltage source!



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Source Transformations

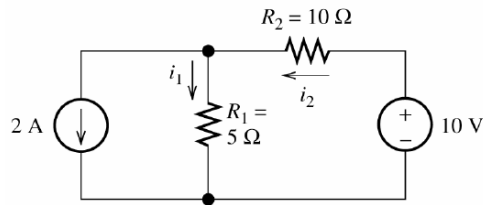


$$KVL: -10V + i_2 R_2 + i_2 R_t - 10V = i_2 10 + i_2 5 - 20V = 0$$

$$i_2 = \frac{20V}{10+5} = 1.33A$$

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Source Transformations



$$i_2 = i_1 + 2A \rightarrow i_1 = i_2 - 2A = 1.333 - 2A = -0.667A$$

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SUPERPOSITION PRINCIPLE

The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually. In equation form, this is

$$r_T = r_1 + r_2 + \cdots + r_n$$

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Superposition Principle

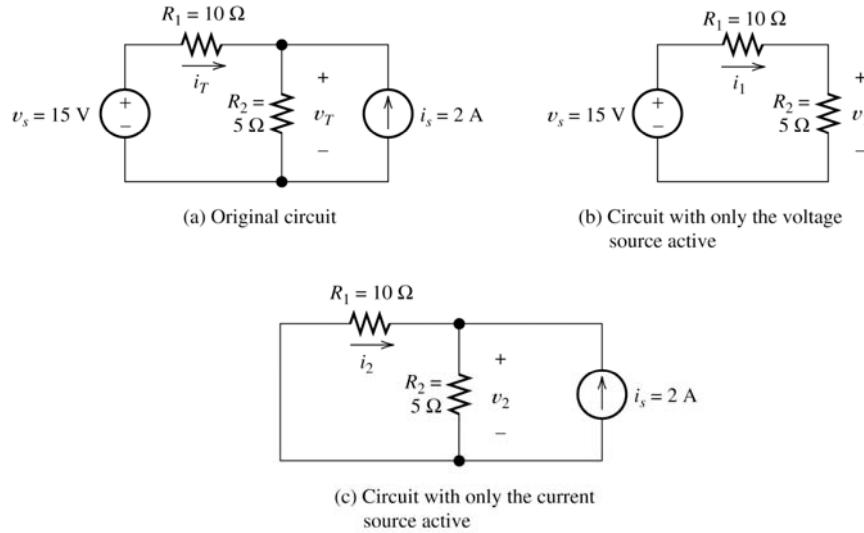
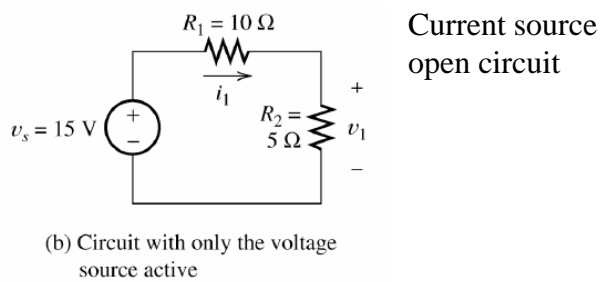


Figure 2.60 Circuit for Example 2.20 and Exercise 2.27.

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Superposition Principle

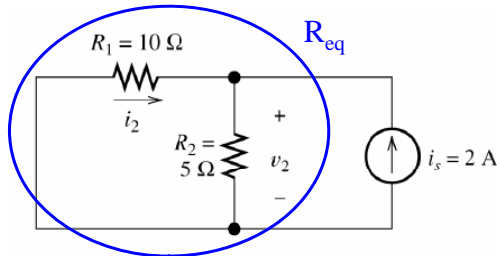


$$v_1 = \frac{R_2}{R_1 + R_2} v_s = \frac{5}{5 + 10} 15V = 5V$$

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Superposition Principle

Voltage source
short circuit



(c) Circuit with only the current source active

$$v_2 = i_s R_{eq} = i_s \frac{R_1 R_2}{R_1 + R_2} = (2A) \frac{(10)(5)}{10 + 5} = (2A)(3.33\Omega) = 6.66V$$

$$v_T = v_1 + v_2 = 5V + 6.66V = 11.66V$$

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Superposition Principle

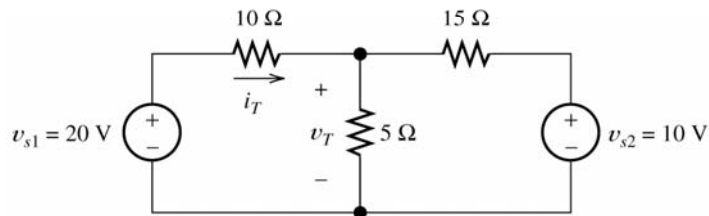
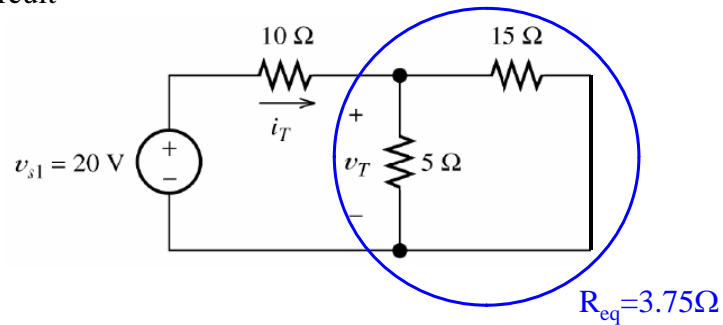


Figure 2.61 Circuit for Exercise 2.28.

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Circuit with just v_{s1}

v_{s2} short circuit

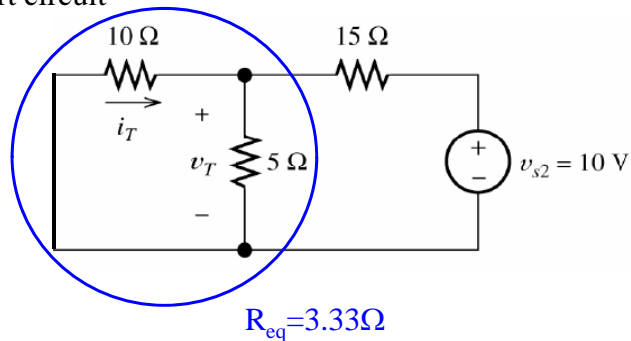


$$v_{T1} = \frac{R_{eq}}{R_{eq} + 10} v_{s1} = \frac{3.75}{3.75 + 10} 20 = 5.45V$$

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Circuit with just v_{s2}

v_{s1} short circuit

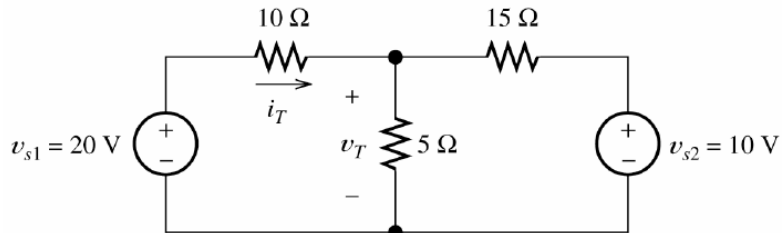


$$v_{T2} = \frac{R_{eq}}{R_{eq} + 15} v_{s2} = \frac{3.33}{3.33 + 15} 10 = 1.82V$$

$$v_T = v_{T1} + v_{T2} = 5.45 + 1.82 = 7.27V$$

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Superposition Principle



$$i_T = \frac{v_{s1} - v_T}{10\Omega} = \frac{20V - 7.27V}{10\Omega} = 1.27A$$

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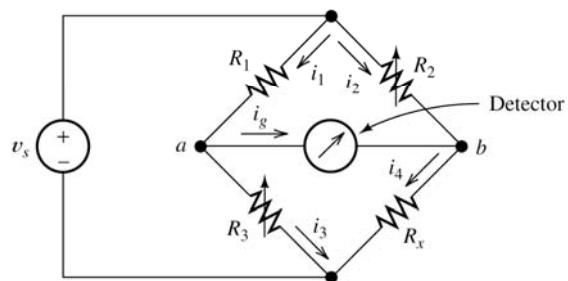


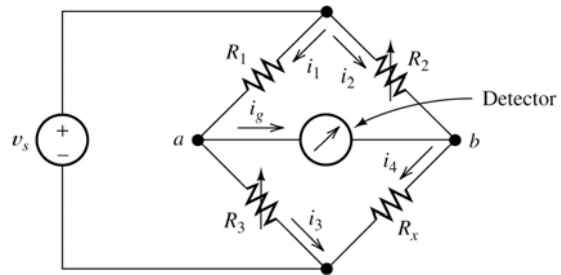
Figure 2.62 The Wheatstone bridge. When the Wheatstone bridge is balanced, $i_g = 0$ and $v_{ab} = 0$.

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WHEATSTONE BRIDGE

The Wheatstone bridge is used by mechanical and civil engineers to measure the resistances of strain gauges in experimental stress studies of machines and buildings.

$$R_x = \frac{R_2}{R_1} R_3$$



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